

**Year 12 Mathematics Specialist 3/4
Test 2 2022**

* Scientific Calculator ONLY
Functions and Sketching Graphs

STUDENT'S NAME _____

DATE: Thursday 24 March

TIME: 50 minutes

MARKS: 52

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three **scientific calculators**, notes on **both sides** of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (5 marks)

The functions f and g are defined by $f(x) = 7x - 1$ and $g(x) = \frac{4}{x - 2}$.

(a) Solve for x if $f \circ g(x) = x$. [3]

(b) Determine the largest value of a such that $g(a) = f^{-1}(a)$. [2]

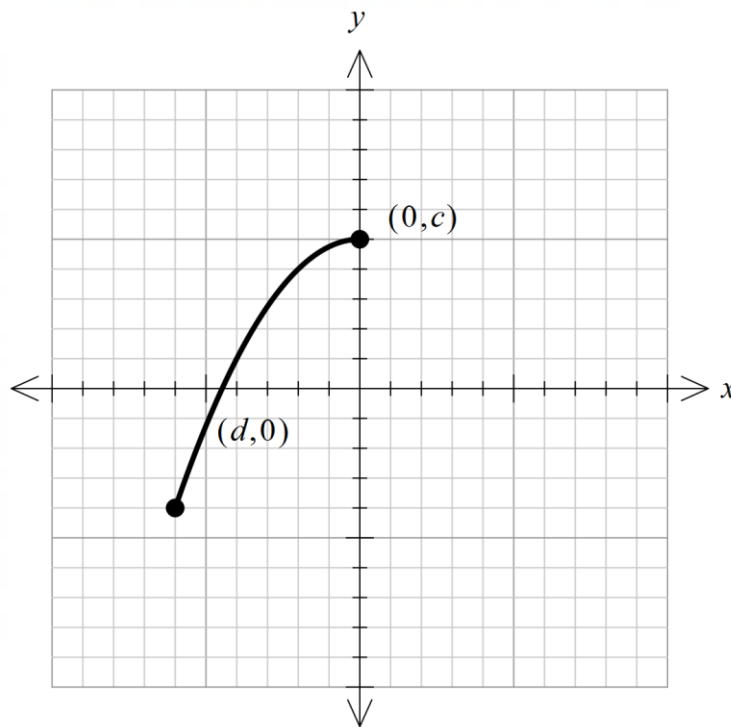
2. (6 marks)

(a) Determine the **two** discontinuities associated with the graph of the function

$$f(x) = \frac{x^2 - 4x + 3}{x^2 - x - 6}.$$

[3]

(b) The graph below shows a sketch of the curve with equation $y = g(x)$, $x \leq 0$. The curve has intercepts at $(0, c)$ and $(d, 0)$.



(i) Explain why $g(x)$ has an inverse function $g^{-1}(x)$.

[1]

(ii) Sketch the graph of $g^{-1}(x)$ on the axes above clearly indicating the coordinates of the x and y intercepts.

[2]

3. (7 marks)

(a) For $f(x) = \sqrt{9+4x^2}$ and $g(x) = \frac{1}{x}$, determine the domain and range of $f \circ g(x)$. [3]

(b) A rational function P is defined by $P(x) = \frac{ax+b}{x+c}$. The graph of $P(x)$ has the following features:

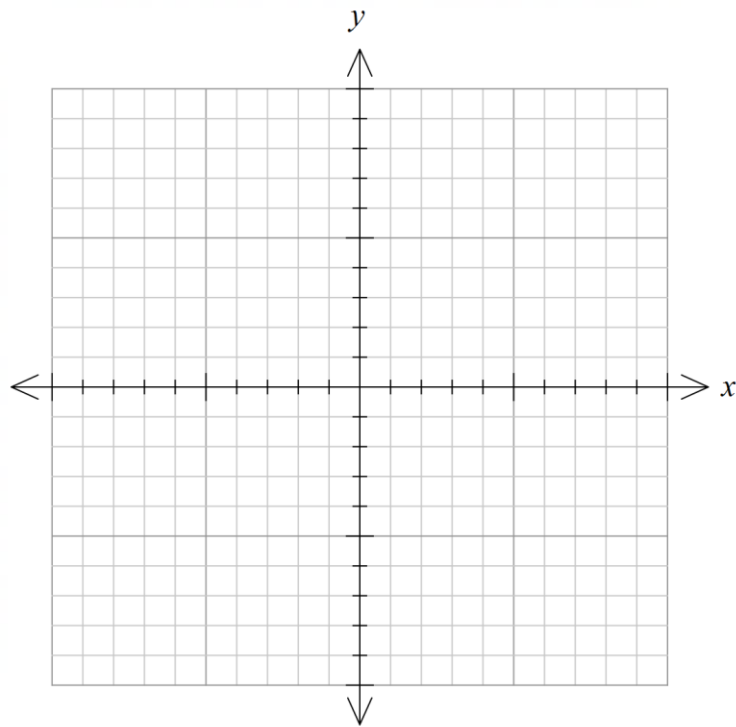
- An x -intercept at $x = -\frac{1}{2}$
- A horizontal asymptote of $y = 2$
- A vertical asymptote of $x = 5$

Determine the values of a , b and c .

[4]

4. (7 marks)

- (a) Sketch the graph of $y = |2x + a|$, $a > 0$ on the axes below showing the coordinates of the points where the graph meets the coordinate axes. [2]



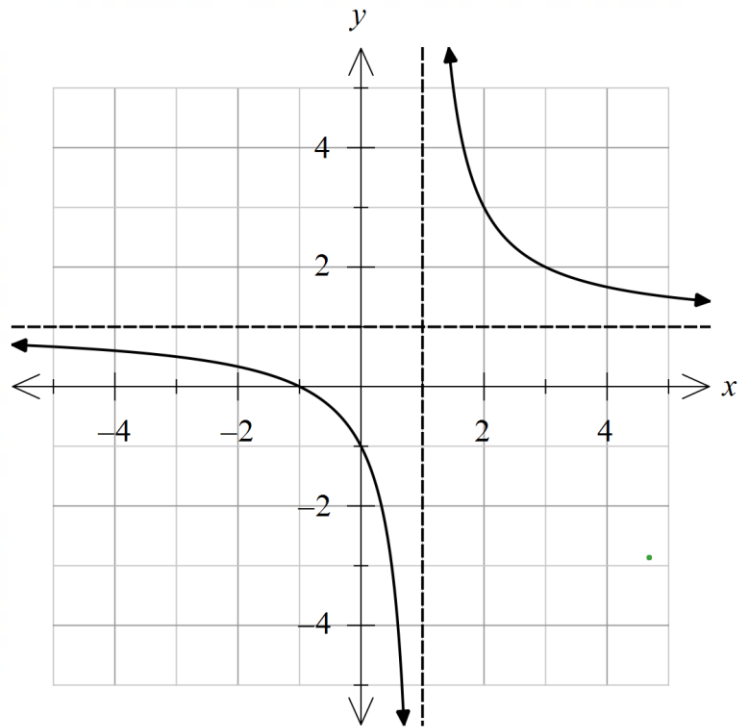
- (b) On the same axes, sketch the graph of $y = \frac{1}{x}$. [1]

- (c) Explain how your graphs show that there is only one solution to the equation $x|2x + a| - 1 = 0$. [1]

- (d) Determine the value of x for which $x|2x + 1| - 1 = 0$. [3]

5. (5 marks)

The graph of $y = f(x) = \frac{x+1}{x-1}$ is drawn on the axes below.



- (a) Sketch on the axes above the graph of $y = |f(x)|$. [2]
- (b) Sketch on the same axes the graph of $y = |x+2|$. [2]
- (c) Hence, state the number of solutions to $\left| \frac{x+1}{x-1} \right| = |x+2|$. You are not required to calculate the solutions. [1]

6. (14 marks)

The curve C has equation $f(x) = \frac{(x-1)^2}{x+1}$.

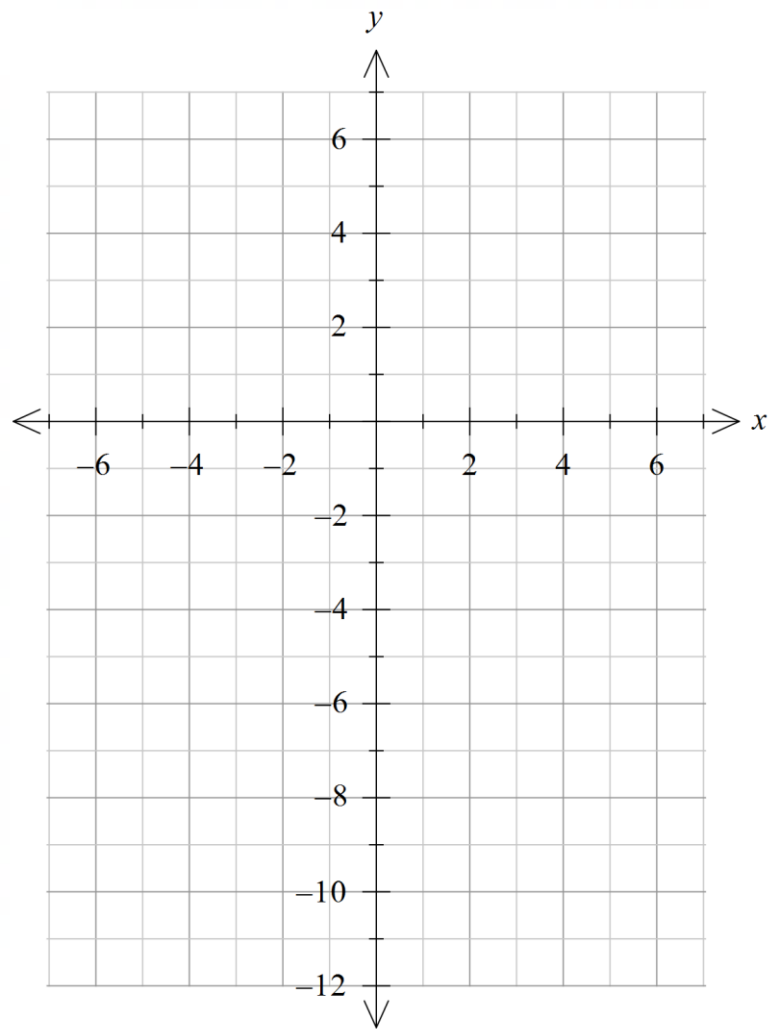
(a) Determine the equations of the asymptotes of C . [3]

(b) Determine the intercepts of C . [2]

(c) Show that C has two stationary points. Determine their coordinates and nature. [4]

(d) Draw a sketch of C on the axes below.

[2]

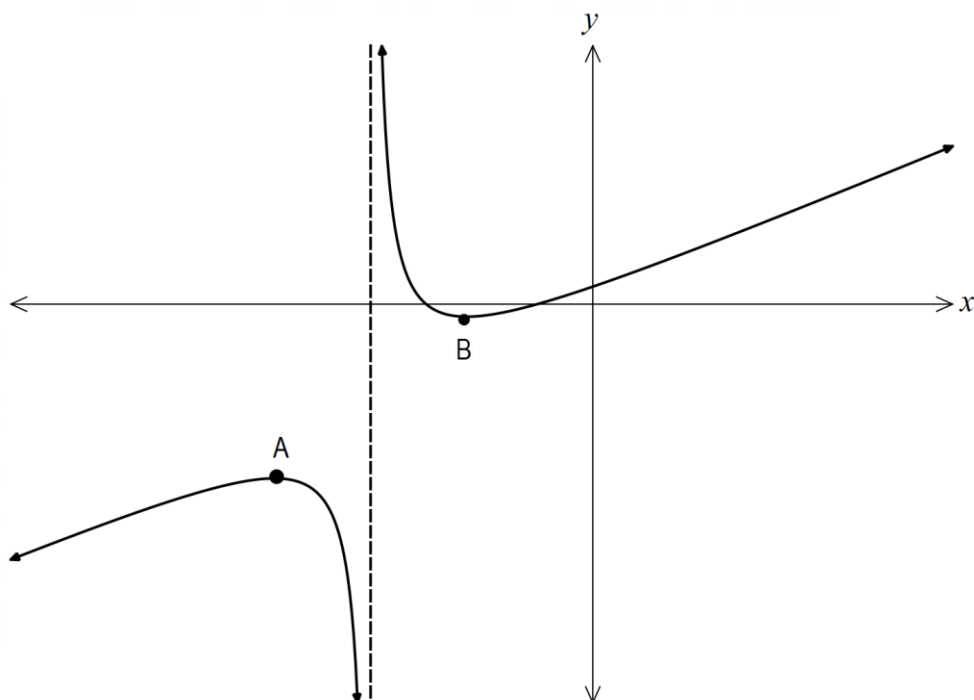


(e) On the same axes above draw a sketch of the curve $y = -f(|x|) + 2$.

[3]

7. (7 marks)

The graph below is a sketch of $y = \frac{(x+1)(x+m)}{x+4}$.



(a) Determine the equation of the vertical asymptote. [1]

(b) Determine the coordinates of the three points where the graph cuts the axes. [3]

- (c) Given that the points A and B are the only stationary points on the curve, determine any restrictions on the value of m . Justify your answer. [3]

[Hint: The function can be written as $y = x + (m - 3) + \frac{12 - 3m}{x + 4}$.]